SPHERICAL AND CHROMATIC ABERRATION IN A LENS:

According to geometric optics, the image of a point object formed in a lens is a point image. In reality, the image of a point object is not a point image, but it is spread in to a region in space both along and perpendicular to the axis of the lens. *The deviation of an optical image in size, shape and position formed by a lens is known as* ***aberration of an optical image***.

The aberration of an image is not due to any defect in the construction of the lens, but it is due to the reasons mentioned below:

(1) The phenomenon of refraction in the lens and

(2) Variation of refractive index of the material of a lens with the wavelength of light.

Monochromatic aberration:

The aberration of optical image even when monochromatic light is used is known as ***monochromatic aberration***.

There are five different types of monochromatic aberrations. They are,

(1) Spherical aberration
(2) Coma
(3) Astigmatism
(4) Curvature of the field and
(5) Distortion

Chromatic aberration:

Aberration of optical images formed in a lens due to the variation of refractive index with the wavelength of light is known as ***chromatic aberration***.

Spherical aberration and its minimization:

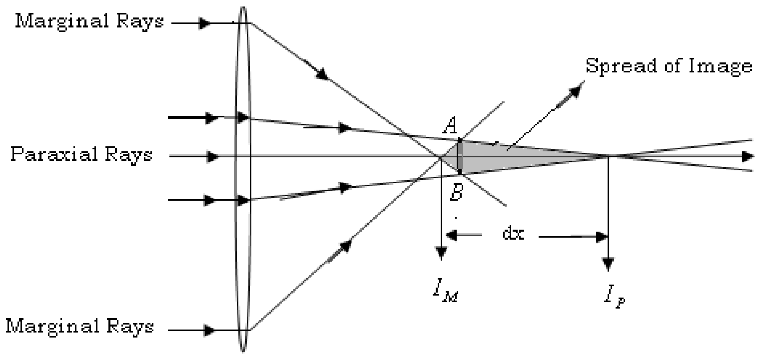


Figure (1) Spherical aberration in convex lens

The rays of light from the distant object after passing through the lens at the margin of the lens [known as marginal rays] converge at a point I_m close to the lens. Similarly, the rays of light passing through a region close to the axis [known as paraxial rays] converge at a point I_p, away from the lens. This results in an image that spreads into a region from I_m to I_p along the axis and from A to B perpendicular to the axis. This defect of the image due to the rays passing through different section of the lens, even with monochromatic light, is known as **spherical aberration of the lens.**

The spread of the image along the axis, [*dx*] is known as **longitudinal spherical aberration.** The image formed at AB is a circle with least diameter and at this position the best image is formed. This circle is called **the circle of least confusion.** The radius of the circle of least confusion measures **lateral spherical aberration.**

Note: The spherical aberration in a convex lens is taken to be positive as the marginal image is formed near the lens than the paraxial image. In the case of concave lens the spherical aberration is taken to be negative as the marginal image is formed to the right side of the paraxial image.

Methods of reducing spherical aberration:

(1) **By using stops:** In this case, the stops used will either allow the paraxial rays or marginal rays. Usually the stop is used to avoid the marginal rays. This brings paraxial and marginal images close to one another thereby reducing the spherical aberration.

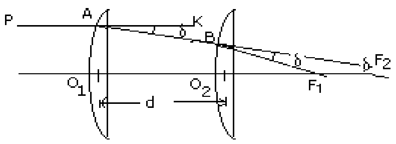
(2) **By the use of Plano-convex lens:** In a lens, the deviation produced by the lens is minimum, when the deviation is shared equally between the two surfaces of the lens. This is achieved in a Plano-convex lens by arranging convex side facing the incident or emergent rays whichever are more parallel to the axis as shown in the following figure(2)

(3) **By the use of crossed lenses:** It is theoretically known that the lenses have minimum spherical aberration when the parallel rays fall of the lens having their radii of curvature r_1 and r_2 bearing a ratio, which satisfies the following condition.

$$\frac{r_1}{r_2} = \frac{\mu(2\mu - 1) - 4}{\mu(2\mu + 1)} \text{ - - - - (1)}$$

In the above equation, μ is the refractive index of the material of the lens. For a lens of μ = 1.5, the ratio $\frac{r_1}{r_2} = -\frac{1}{6}$. A lens having its radii of curvature satisfying this condition is known as a **crossed lens**.

(4) **By using two Plano-convex lenses separated by a suitable distance:** When the two plano-convex lenses are separated at a suitable distance, the total deviation is divided equally between the two lenses and the total deviation is minimum. This reduces the spherical aberration to minimum. The necessary condition is derived as follows.



With reference to figure (4), we can write,

$\angle BAK = \angle BF_2O_2 = \delta$, Also, $\angle F_1BF_2 = \angle BF_2\,F_1 = \delta$,

So that $F_1F_2 = F_1B = F_1O_2$ Or $O_2F_1 = ½\, O_2F_2$.

Since F_2 is the virtual object of the real image F_1 and using the lens formula for the second lens, we can write the equation, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f_2}$ - - (2)

In this equation $u = f_1 - d$ & $v = \frac{f_1 - d}{2}$ - - (3)

Substituting for 'u' and 'v' and simplifying, we get,

$$\frac{2}{f_1 - d} - \frac{1}{f_1 - d} = \frac{1}{f_2} \Rightarrow \frac{1}{f_1 - d} = \frac{1}{f_2} \Rightarrow f_2 = f_1 - d$$

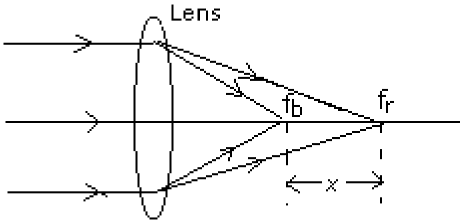
$$\text{Or, } d = f_1 - f_2 \text{ - - - (4)}$$

Equation (4) gives the condition for minimum spherical aberration.

(5) By using suitable concave and convex lenses in contact: Since spherical aberration produced by convex lens is positive and that produced by a concave lens is negative, a suitable combination of convex and concave lens will minimize the spherical aberration.

Longitudinal or axial chromatic aberration:

When a parallel beam of white light is passed through a lens, blue rays are brought to focus at a point near the lens and red rays are brought to focus at a point away from the lens and other coloured foci are formed in between them. Thus, the image spread over a distance 'x' from blue focus to red focus and this distance x = $f_r - f_b$, is called the longitudinal or axial chromatic aberration. An equation for axial chromatic aberration is derived as follows.



The focal length of a lens is given by,

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \text{ - - - } (1)$$

$$\text{Or, } \left[\frac{1}{R_1} - \frac{1}{R_2}\right] = \frac{1}{f(\mu - 1)} \text{ - - - } (2)$$

Similarly, the focal length for the blue and red rays is given by,

$$\frac{1}{f_b} = (\mu_b - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = \frac{(\mu_b - 1)}{(\mu - 1) f} \text{ - - - - } (3)$$

$$\text{Also, } \frac{1}{f_r} = (\mu_r - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = \frac{(\mu_r - 1)}{(\mu - 1) f} \text{ - - - - } (4)$$

Subtracting equation (4) from equation (3), we get,

$$\frac{1}{f_b} - \frac{1}{f_r} = \frac{(\mu_b - 1 - \mu_r + 1)}{(\mu - 1) f} \Rightarrow \frac{f_r - f_b}{f_r f_b} = \frac{(\mu_b - \mu_r)}{(\mu - 1) f} \text{ - - - - } (5)$$

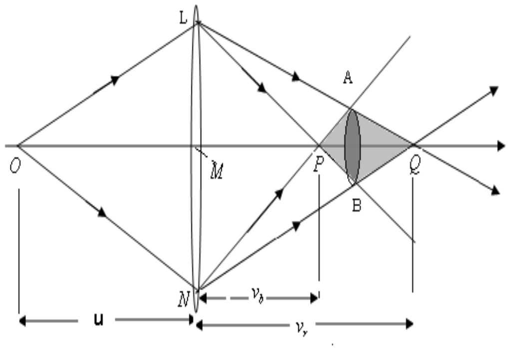
Taking $f_r f_b = f^2$ (where f is the mean focal length), we can write equation (5) as

$$\frac{f_r - f_b}{f^2} = \frac{(\mu_b - \mu_r)}{(\mu - 1) f} \Rightarrow f_r - f_b = \left[\frac{(\mu_b - \mu_r)}{(\mu - 1)}\right] f = \omega \times f$$

$$\text{Thus, } f_r - f_b = \omega f \text{ - - - - } (6)$$

In equation (6) 'ω' is the dispersive power of the material of the lens and f is the focal length of the mean ray. Therefore, axial chromatic aberration is equal to the product of the dispersive power of the material of the lens and the focal length of the lens. As ω and f are constant for a lens, a single lens cannot be used to minimize axial chromatic aberration. As a concave lens forms virtual focus, the focal length of the lens for mean ray is negative and hence a suitable combination of a convex and a concave lens can minimize axial chromatic aberration.

Circle of least confusion, a measure of lateral chromatic aberration:



Let a point object illuminated by white light is situated on the axis at a distance "u" from the lens and the blue and red images are formed on the axis at positions P and Q such that the coloured images spread from P to Q. A screen placed at AB has an image with least lateral chromatic aberration. The diameter of the circle of least confusion gives a measure of lateral chromatic aberration and equation for is calculated as follows.

Let 'u' be the object distance and 'v_r' and 'v_b' denote the image distance for red and blue images. If f_r and f_b represent the focal lengths for the red and blue rays of light, then,

$$\frac{1}{v_b}-\frac{1}{u}=\frac{1}{f_b} \quad \text{- - - - (1)}$$

$$\text{and } \frac{1}{v_r}-\frac{1}{u}=\frac{1}{f_r} \quad \text{- - - - - - (2)}$$

Subtracting equation (2) from (1), we get,

$$\frac{1}{v_b}-\frac{1}{v_r}=\frac{1}{f_b}-\frac{1}{f_r} \Rightarrow \frac{v_r-v_b}{v_r v_b}=\frac{f_r-f_b}{f_r f_b}$$

Taking $v_r v_b = v^2$ and $f_r f_b = f^2$, the above equation becomes,

$$\frac{v_r-v_b}{v^2}=\frac{f_r-f_b}{f^2}=\frac{wf}{f^2}=\frac{w}{f} \quad \text{[Because, } f_r - f_b = w f\text{]}$$

$$\text{Therefore, } v_r - v_b = \frac{wv^2}{f} \quad \text{- - - - (3)}$$

From similar triangles LQN and AQB we can write, $\frac{CQ}{AB}=\frac{MQ}{LN}$ - - - - (4)

Also, from similar triangles LPN and APB we can write, $\frac{CQ}{AB}=\frac{MP}{LN}$ - - - - (5)

Adding equations (4) and (5), we get, $\frac{PC+CQ}{AB}=\frac{MQ+MP}{LN} \Rightarrow \frac{PQ}{AB}=\frac{MQ+MP}{LN}$ - - - - (6)

But, PQ = $v_r - v_b$; AB = d, in the diameter of the circle of least confusion and LN = D is the diameter of the lens aperture and MQ + MP = $v_r + v_b = 2v$ approximately. Substituting these values in equation (6), we get, $\frac{v_r-v_b}{d}=\frac{2v}{D} \Rightarrow d = D\times\left[\frac{(v_r-v_b)}{2v}\right]$

Using equation (3), we can write, $d = D\times\left[\frac{1}{2v}\right]\frac{wv^2}{f}=\frac{1}{2}\cdot Dw\cdot\frac{v}{f}$ - - - - - (7)

For a parallel beam of incident light, $v = f$ and hence equation (7) reduces to the form,

$$d == \tfrac{1}{2}\cdot Dw\cdot\frac{f}{f}=\tfrac{1}{2}\cdot D\cdot w \quad \text{- - - - - (8)}$$

Thus, the lateral chromatic aberration depends on the diameter of the lens aperture and the dispersive power of the material, but it is independent of the focal length of the lens.

Condition for achromatism of two lenses placed in contact:

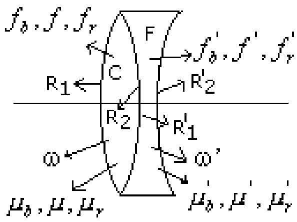


Fig (1)

Let a convex lens C made of crown glass and a concave lens F made of flint glass in contact act as achromatic combination. Let μ_b, μ, μ_r and μ'_b, μ', μ'_r represent the refractive indices for blue, yellow and red rays of light of the two materials of the lenses. Let f_b, f, f_r and f'_b, f', f'_r are corresponding focal lengths of the two lenses and ω and ω' are the dispersive powers for crown and flint glass respectively. Let R_1 and R_2 be the radii of curvature of the Crown glass lens and let R'_1 and R'_2 be the radii of curvature of the Flint glass lens.

Then using lens maker's formulae, we can write for the Crown glass lens,

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \Rightarrow \left[\frac{1}{R_1} - \frac{1}{R_2}\right] = \frac{1}{(\mu-1)f} \quad \text{--- (1)} \qquad \text{[for yellow ray]}$$

$$\frac{1}{f_r} = (\mu_r - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \Rightarrow \frac{1}{f_r} = \frac{(\mu_r - 1)}{(\mu-1)f} \quad \text{----- (2)} \qquad \text{[for red ray]}$$

$$\text{Also, } \frac{1}{f_b} = (\mu_b - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \Rightarrow \frac{1}{f_b} = \frac{(\mu_b - 1)}{(\mu-1)f} \quad \text{----- (3)} \qquad \text{[for ray]}$$

In the same way, for the Flint concave lens, we can write,

$$\frac{1}{f'} = (\mu' - 1)\left[\frac{1}{R'_1} - \frac{1}{R'_2}\right] \Rightarrow \left[\frac{1}{R'_1} - \frac{1}{R'_2}\right] = \frac{1}{(\mu'-1)f'} \quad \text{---- (4)} \qquad \text{[for yellow ray]}$$

$$\frac{1}{f'_r} = (\mu_r - 1)\left[\frac{1}{R'_1} - \frac{1}{R'_2}\right] \Rightarrow \frac{1}{f'_r} = \frac{(\mu'_r - 1)}{(\mu'-1)f'} \quad \text{----- (5)} \qquad \text{[for red ray]}$$

$$\frac{1}{f'_b} = (\mu_b - 1)\left[\frac{1}{R'_1} - \frac{1}{R'_2}\right] \Rightarrow \frac{1}{f'_b} = \frac{(\mu'_b - 1)}{(\mu'-1)f'} \quad \text{---- (6)} \qquad \text{[for blue ray]}$$

If F_b and F_r denote the focal lengths of the combination for blue and red rays o flight, then, we can write,

$$\frac{1}{F_r} = \frac{1}{f_r} + \frac{1}{f'_r} = \frac{(\mu_r - 1)}{(\mu-1)f} + \frac{(\mu'_r - 1)}{(\mu'-1)f'} \quad \text{---- (7)}$$

$$\text{Similarly, } \frac{1}{F_b} = \frac{1}{f_b} + \frac{1}{f'_b} = \frac{(\mu_b - 1)}{(\mu-1)f} + \frac{(\mu'_b - 1)}{(\mu'-1)f'} \quad \text{---- (8)}$$

For the combination to be achromatic, the focal lengths F_b and F_r must be equal. Thus,

$$F_r = F_b \text{ or } \frac{1}{F_r} = \frac{1}{F_b} \Rightarrow \frac{(\mu_r - 1)}{(\mu-1)f} + \frac{(\mu'_r - 1)}{(\mu'-1)f'} = \frac{(\mu_b - 1)}{(\mu-1)f} + \frac{(\mu'_b - 1)}{(\mu'-1)f'}$$

$$\Rightarrow \frac{(\mu_b - 1)}{(\mu-1)f} - \frac{(\mu_r - 1)}{(\mu-1)f} + \frac{(\mu'_b - 1)}{(\mu'-1)f'} - \frac{(\mu'_r - 1)}{(\mu'-1)f'} = 0 \Rightarrow \frac{(\mu_b - \mu_r)}{(\mu-1)f} + \frac{(\mu'_b - \mu'_r)}{(\mu'-1)f'} = 0$$

Using $\frac{(\mu_b - \mu_r)}{(\mu-1)}$ = ω and $\frac{(\mu'_b - \mu'_r)}{(\mu'-1)}$ = ω', we get, $\frac{\omega}{f} + \frac{\omega'}{f'} = 0$; Or $\frac{\omega}{f} = -\frac{\omega'}{f'} \Rightarrow \boxed{f' = -f\frac{\omega'}{\omega}}$ ---- (9)

Since ω and ω' are positive quantities, f' is negative if f is positive. Thus if crown glass is used to make convex lens, then flint glass lens must be concave. The ratio of the dispersive powers of the material of the lenses must be equal to the ratio of the focal lengths of the two lenses.

CONDITION FOR ACHROMATISM OF TWO THIN LENSES SEPARATED BY FINITE DISTANCE:

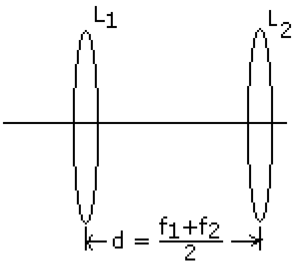


Figure (2) Achromatism of two thin lenses separated by a distance

Let f_1 and f_2 be the two convex lenses separated by a distance 'd' such that they act as achromatic combination. Let the two lenses are made of the same material and let μ,μ_b, and μ_r denote the refractive indices for the mean ray, blue rays and red rays respectively. Let f_r, f_r' and f_b, f_b' are the focal lengths of the two lenses for red and blue rays of light.

Then, the equivalent focal length of the two lenses for mean ray, red ray and blue ray are respectively given by the following equations

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \text{ - - - - (1) [for mean ray]} \qquad \frac{1}{F_r} = \frac{1}{f_r} + \frac{1}{f_r'} - \frac{d}{f_r f_r'} \text{ - - - - (2) [for red rays]}$$

$$\frac{1}{F_b} = \frac{1}{f_b} + \frac{1}{f_b'} - \frac{d}{f_b f_b'} \text{ - - - - - (3) [for blue rays]}$$

In the above equations, F, F_r and F_b are the combined focal length for the mean rays, red rays and blue rays.

But $\frac{1}{f_r} = \frac{(\mu_r - 1)}{(\mu - 1) f_1}, \frac{1}{f_r'} = \frac{(\mu_r - 1)}{(\mu - 1) f_2}$ & $\frac{1}{f_b} = \frac{(\mu_b - 1)}{(\mu - 1) f_1}, \frac{1}{f_b'} = \frac{(\mu_b - 1)}{(\mu - 1) f_2}$ - - - - (4)

Using equation (4) in equations (2) and (3), we get,

$$\frac{1}{F_r} = \frac{(\mu_r - 1)}{(\mu - 1) f_1} + \frac{(\mu_r - 1)}{(\mu - 1) f_2} - \frac{(\mu_r - 1)^2}{(\mu - 1)^2} \frac{d}{f_1 f_2} \text{ - - - - (5)}$$

and $$\frac{1}{F_b} = \frac{(\mu_b - 1)}{(\mu - 1) f_1} + \frac{(\mu_b - 1)}{(\mu - 1) f_2} - \frac{(\mu_b - 1)^2}{(\mu - 1)^2} \frac{d}{f_1 f_2} \text{ - - - - (6)}$$

For the combination to be achromatic, $F_r = F_b$; Or $\frac{1}{F_r} = \frac{1}{F_b}$

Using equation (5) and (6), we can write,

$$\frac{(\mu_r - 1)}{(\mu - 1) f_1} + \frac{(\mu_r - 1)}{(\mu - 1) f_2} - \frac{(\mu_r - 1)^2}{(\mu - 1)^2} \frac{d}{f_1 f_2} = \frac{(\mu_b - 1)}{(\mu - 1) f_1} + \frac{(\mu_b - 1)}{(\mu - 1) f_2} - \frac{(\mu_b - 1)^2}{(\mu - 1)^2} \frac{d}{f_1 f_2}$$

$$\frac{(\mu_r - 1)}{(\mu - 1)}\left[\frac{1}{f_1} + \frac{1}{f_2}\right] - \frac{(\mu_r - 1)^2}{(\mu - 1)^2} \frac{d}{f_1 f_2} = \frac{(\mu_b - 1)}{(\mu - 1)}\left[\frac{1}{f_1} + \frac{1}{f_2}\right] - \frac{(\mu_b - 1)^2}{(\mu - 1)^2} \frac{d}{f_1 f_2}$$

Re arranging, the above equation, we get,

$$\frac{(\mu_b - \mu_r)}{(\mu - 1)}\left[\frac{1}{f_1} + \frac{1}{f_2}\right] = \frac{d}{(\mu - 1)^2 f_1 f_2}\left((\mu_b - 1)^2 - (\mu_r - 1)^2\right) = \frac{d}{(\mu - 1)^2 f_1 f_2}(\mu_b - \mu_r)[\mu_b + \mu_r - 2]$$

$$= \frac{d}{(\mu - 1)^2 f_1 f_2}(\mu_b - \mu_r)[2\mu - 2] \quad \{\text{Taking } \mu_b + \mu_r = 2\mu\}$$

$$\frac{(\mu_b - \mu_r)}{(\mu - 1)}\left[\frac{1}{f_1} + \frac{1}{f_2}\right] = \frac{d}{(\mu - 1)^2 f_1 f_2}(\mu_b - \mu_r)2[\mu - 1] = \frac{2d(\mu_b - \mu_r)}{(\mu - 1) f_1 f_2}$$

Thus, we get, $\frac{1}{f_1} + \frac{1}{f_2} = \frac{2d}{f_1 f_2}$ **or** $\frac{f_1 + f_2}{f_1 f_2} = \frac{2d}{f_1 f_2}$ $\Rightarrow$ $d = \frac{f_1 + f_2}{2}$ - - - - - (7)

Equation (7) gives the condition for the two thin convex lenses separated by a distance for them to act as achromatic combination of lenses.

CARDINAL POINTS OF AN OPTICAL SYSTEM

Gauss introduced the concept of *"cardinal points of an optical system"* in 1841. Gauss showed that any number of co-axial refracting systems can be treated as one unit and the simple formulae for thin lenses can be applied provided the distances are measured from two theoretical parallel planes fixed with reference to the refracting system. The points of intersection of these planes with the axis are called the *principal points* or *Gauss points*. Actually there are six points [three pair] in all, which are important in understanding the refraction through a co-axial lens system. Of these, two are *principal points*; two are *principal fici* or focal points and two are *nodal points*. These six points are together known as "***cardinal points of an optical system***".

Therefore, *cardinal points of an optical system is a group of three pairs [six] points such that when different distances are measured from these fixed pair of points, the same formulae for thin lenses can be used for any optical system.*

Principal foci (focal points) and focal planes:

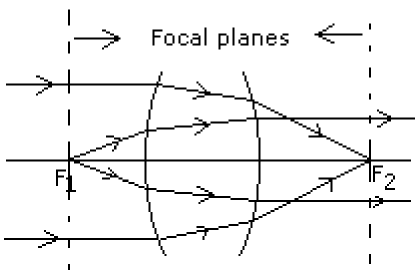


Figure (1a)

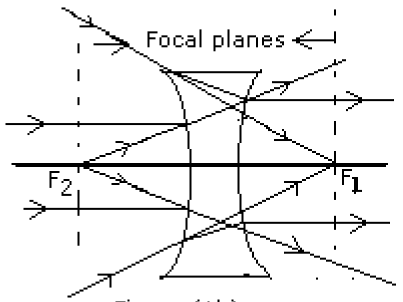


Figure (1b)

A set of rays incident on the system of lenses parallel to the axis after passing through the lens system converges at a point [converging system] or appear to diverge from a fixed point F_2 on the axis. This point is known as the second principal focus. In other words, the position of the image on the axis corresponding to an axial point object at infinity is known as the *second focal point* of the lens. A plane intersecting the second focal point and perpendicular to the axis of the lens system is known as the second focal plane.

Similarly, if a set of rays from a fixed point [in the case of converging system] on the axis after passing through the lens system or directed towards a fixed point F_1 [in the case of a diverging system] become parallel to the axis after passing through the lens system. This point is known as *the first focal point* or *first principal focus*. In other words, the first principal focus corresponds to the position of an axial point object on the axis for which the final image is formed at infinity. A plane intersecting the first focal point and perpendicular to the axis is known as the first focal plane of the lens system. Figure (1a) and (1b) shows the position of focal points and focal planes of a lens system.

Principal points and principal planes:

The first principal plane in the object space is the locus of the points of intersection of the emergent rays in the image-space parallel to the axis and their conjugate incident rays in the object space. The second principal plane in the image space is the locus of the point s of intersection of the incident rays in the object space parallel to the axis and their conjugate emergent rays in the image space.

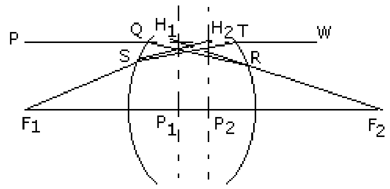


Figure (2)

Let us consider a thick lens or co-axial lens system, having the principal foci at F_1 and F_2 as shown in figure (2). The ray incident at the point Q and parallel to the axis, after refraction through the lens takes the direction RF_2 passing through the second principal focus F_2. The incident and the emergent rays when produced intersect at H_2. A plane passing through H_2 and perpendicular to the axis is termed as the second principal plane of the lens. The point of intersection P_2 with the axis is known as the second principal point.

Consider another ray F_1S passing through the first principal focus F_1 such that after refraction it emerges along TW parallel to the axis at the same height as the incident ray at Q. The emergent ray TW and the corresponding incident ray F_1S produced meet at a point H_1 as shown in figure (2). A plane perpendicular to the axis and passing through H_1 is known as the ***first principal plane*** and its point of intersection P_1 with the axis is called the ***first principal point***.

Note: Two rays of light directed towards a point H_1 on the first principal plane appear to start from a point H_2 on the second principal plane. Therefore, H_2 is the image of H_1. Thus the points H_1 and H_2 are called conjugate points and the planes H_1P_1 and H_2P_2 are known as conjugate planes. The heights $H_1P_1 = H_2P_2$ and hence these planes have a lateral magnification of +1.

Nodal points and Nodal planes:

Nodal points are defined as a pair of conjugate points on the axis having unit positive angular magnification. This means that a ray of light directed towards one of these points, after refraction through the lens appears to proceed from the second point in a direction parallel to the incident direction as shown in figure (3).

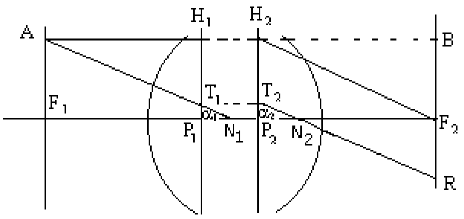


Figure (3)

Let H_1P_1 and H_2P_2 be the first and the second principal plane of an optical system and let AF_1 and BF_2 represent its first and second focal planes respectively. A ray of light AH_1 from a point 'A' on the first focal plane incident parallel to the axis proceeds along the direction H_2F such that $H_1P_1 = H_2P_2$.

Another incident ray AT_1 parallel to the emergent ray H_2F_2 and strikes the principal plane at T_1. The corresponding emergent ray start from a point T_2 such that $T_2P_2 = T_1P_1$ and will proceed parallel to the ray H_2F_2 as the two incident rays originate from the same point A on the first focal plane of the lens system. Then, the point of intersection of the incident ray AT_1 on the axis give the position of the first nodal point N_1 and the point of intersection of the conjugate emergent ray T_2R with the axis give the position of the second nodal point N_2.

Properties of nodal points:

(1) Nodal points have unit angular magnification, because, $\tan \alpha_1 = \tan \alpha_2$

(2) Distance $P_1N_1 = P_2N_2$. The distance between the principal point and the nodal points are same.

(3) Distance $P_1P_2 = N_1N_2$. The distance between the two principal points is equal to the distance between the two nodal points.

(4) Distance $P_1N_1 = P_2N_2 = P_2F_2 - F_1P_1 = f_1 + f_2$, where f_1 is positive and f_2 is negative.

(5) If the medium on either side of the lens system is the same, then $f_1 = -f_2$ and hence nodal points coincides with the corresponding principal points.

Relation between the distance of the different cardinal points of a lens system and the focal length of the lens system in the case of a co-axial lens system:

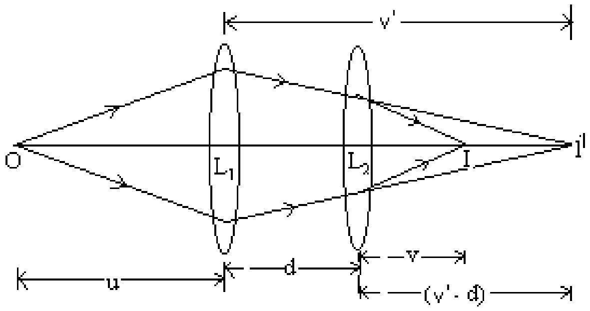


Figure (4)

Consider two thin lenses L_1 and L_2 of focal lengths f_1 and f_2 separated by a distance 'd'. Let u be the distance of the object from the first lens and the final image is formed at I at a distance v from the second lens. Let the first image due to the first lens is formed at I^1.

$$\therefore \frac{1}{v^1} - \frac{1}{u} = \frac{1}{f_1} \text{ or } \frac{1}{v^1} = \frac{1}{f_1} + \frac{1}{u} = \frac{u + f_1}{uf_1} \text{ - - - - (1)}$$

The image I^1 behaves as a virtual object for the second lens and the object distance for the second lens is $(v^1\text{-}d)$ and the final image is formed at I.

$$\frac{1}{v} - \frac{1}{v^1 - d} = \frac{1}{f_2} \quad \text{Or} \quad \frac{1}{v^1 - d} = \frac{1}{v} - \frac{1}{f_2} = \frac{f_2 - v}{vf_2}$$

$$\text{Or, } (v^1 - d) = \frac{vf_2}{f_2 - v}$$

Substituting the value of v^1 from equation (1), we get,

$$\frac{f_1 u}{u + f_1} - d = \frac{vf_2}{f_2 - v}$$

Multiplying by $(u + v)(f_2 - v)$, we get,

$$f_1 u(f_2 - v) - d(u + f_1)(f_2 - v) = vf_2(u + f_1)$$

$$f_1 f_2 u - f_1 uv - duf_2 + duv - df_1 f_2 + df_1 v = uvf_2 + vf_1 f_2$$

$$uv(d - f_1 - f_2) + u(f_1 f_2 - df_2) + v(df_1 - f_1 f_2) - df_1 f_2 = 0 \text{ - - (2)}$$

This equation (2) can be written as

$$uvA + uB + vC + D = 0 \text{ - - - - (3)}$$

In equation (3), A B and C are coefficients. Dividing equation (3) by A, we get,

$$uv + u\frac{B}{A} + v\frac{C}{A} + \frac{D}{A} = 0 \text{ - - - - (4)}$$

Let the focal length of the equivalent lens is "f" and $U = u - \alpha$ [distance of object measured from the first principal plane] and $V = v - \beta$ [distance of the image measured from the second principal plane] denote respectively the reduced object distance and reduced image distance. In these equations α is the distance of the first lens from the first principal plane and β is the distance of the second lens from the second principal plane. Then, the lens formula for the reduced distances is given by,

$$\frac{1}{V}-\frac{1}{U}=\frac{1}{f} \Rightarrow \frac{1}{v-\beta}-\frac{1}{u-\alpha}=\frac{1}{f}$$

Multiplying by (v -β)(u - α)f, we get,

$$(u-\alpha)f-(v-\beta)f=(u-\alpha)(v-\beta)$$

Simplifying and rearranging,

$$uv+u(-\beta-f)+v(-\alpha+f)+(\alpha\beta-\beta f+\alpha f)=0 \text{ - - - - } (5)$$

Comparing equation (4) and (5), we get,

$$-\beta-f=\frac{B}{A} \text{ - - - - } (6)$$

$$-\alpha+f=\frac{C}{A} \text{ - - - - } (7)$$

$$\alpha\beta-\beta f+\alpha f=\frac{D}{A} \text{ - - - - } (8)$$

Multiplying equation (6) and (7),

$$\alpha\beta-\beta f+\alpha f-f^2=\frac{BC}{A^2} \text{ - - - - } (9)$$

Subtracting (9) from (8),

$$f^2=\frac{D}{A}-\frac{BC}{A^2}=\frac{DA-BC}{A^2} \text{ - - - - } (10)$$

Substituting the value of the coefficients A, B, C and D from equation (2), we get,

$$f^2 \frac{(-df_2 f_2)(d-f_1-f_2)-(f_1 f_2-df_2)(df_1-f_1 f_2)}{(d-f_1-f_2)^2}$$

$$f^2 \frac{-d^2 f_2 f_2+df_1^2 f_2+df_1 f_2^2+f_1^2 f_2^2+d^2 f_1 f_2-df_1 f_2^2}{(f_1+f_2-d)^2}$$

$$\text{Or, } f^2=\frac{f_1^2 f_2^2}{(f_1+f_2-d)^2} \text{ or } f=\frac{\pm f_1 f_2}{(f_1+f_2-d)}$$

$$\text{Or, } f=\frac{f_1 f_2}{(f_1+f_2-d)} \text{ or } \frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2} \text{ - - - - } (11)$$

$$\text{Also, } f_1+f_2-d=\frac{f_1 f_2}{f} \text{ - -- - } (12)$$

From equation (7), we can write,

$$\alpha=f-\frac{C}{A}=\frac{f_1 f_2}{f_1+f_2-d}-\frac{-df_1+f_1 f_2}{f_1+f_2-d}=\frac{df_1-f_1 f_2+f_1 f_2}{f_1+f_2-d}=\frac{df_1}{f_1+f_2-d}\ \alpha=\frac{df_1}{f_1+f_2-d} \text{ - - - - } (13)$$

Using equation (12), we can write, $\alpha=\frac{d\times f}{f_2}$ - - - - (14)

In the same way, we can show from equation (6) that, $\beta=\frac{-df_2}{f_1+f_2-d}=\frac{-d\times f}{f_1}$ - - - - - (15)

Also, from equation (14) and (15) we can see that

$$\frac{f_1}{f_2}=\frac{\alpha}{\beta} \text{ - - - - } (16)$$

EYEPIECES

An eyepiece is an arrangement of two lenses separated by a suitable distance and it has the following advantages over a single eye lens.

(i) It increases the angular object field
(ii) It brings the centre of the exit pupil nearer the eye lens and increases the angular image field
(iii) It helps to minimize the chromatic and spherical aberrations.

HUYGENS EYEPIECE:

It consists of two plano-convex lenses having focal lengths in the ratio 3 : 1 and the distance between them is equal to the difference in their focal lengths. If 3f and f are the focal lengths of the two lenses, then d =3f – f = 2f and also, average focal length of the two lenses $\frac{3f+f}{2} = 2f$ = d and hence the condition for minimum spherical aberration and minimum achromatic aberration are satisfied by the lens system. Hence, this eyepiece is free from spherical and chromatic aberration. Image formation in Huygens eyepiece is illustrated in figure (1).

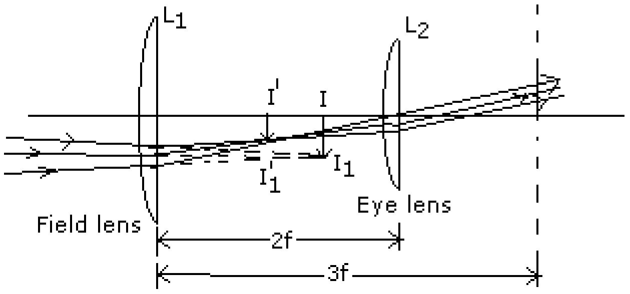
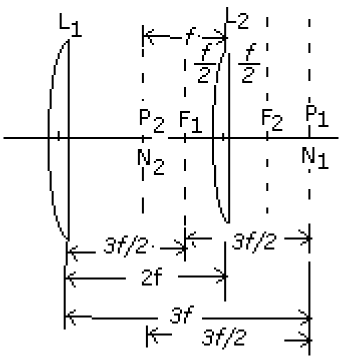


Figure (1)

The image [II_1] of the distant object due to objective is formed at the focal plane of the field lens in its absence. The rays of light incident on the field lens pass through it and form the image at $I_1I_1^1$. This image is formed at the focal plane of the eye lens and the final image is formed at infinity.

Huygens eyepiece is known as a negative eyepiece as the image due to objective is formed behind the field lens. [Object for the eyepiece is virtual.] It is for this reason, in this eyepiece cross-wires cannot be used and hence this eyepiece cannot be used in those instruments in which quantitative study of images are made.

Cardinal points of Huygens Eyepiece:



Focal length of the lens combination [eyepiece] is given by,

$$F = \frac{f_1 f_2}{f_1 + f_2 - d} = \frac{3f \times f}{3f + f - 2f} = \frac{3}{2} f \quad ---- (1)$$

The distance of the first principal point from the field lens is given by,

$$\alpha = \frac{F \times d}{f_2} = \frac{\frac{3}{2} f \times 2f}{f} = 3f \quad ---- (2)$$

Similarly, the distance of the second principal point from the second lens is given by,

$$\beta = -\frac{F\times d}{f_1} = -\frac{\frac{3}{2}f\times 2f}{3f} = -f \text{ - - - - } (3)$$

Thus, the first focal point F_1 is at a distance 3f/2 from the first principal point & hence it is at a distance 3f/2 from the field lens. The second principal point F_2 is at a distance 3f/2 from the second principal point & hence it is at a distance f/2 to the right of the eye lens.

RAMSDEN EYEPIECE:

Ramsden eyepiece consists of two plano-convex lenses of equal focal length separated by a distance equal to two-third of the focal length of either. The two lenses are arranged such that the convex surfaces of the two lenses face each other. [Outer faces are plane.] The eyepiece is placed beyond the position of the image due to the objective lens. In this eyepiece cross wires can be used and hence it can be used for quantitative measurement of the image.

The image formation in Ramsden eyepiece due to refraction through different lenses is as shown in the following figure (2)

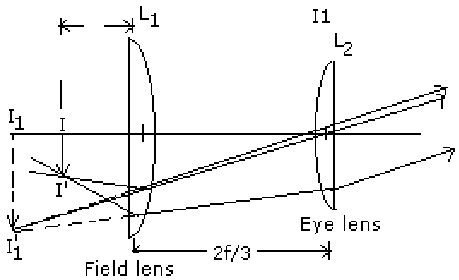


Figure (1)

In this eyepiece, $d \neq (f_1+f_2)/2$ and hence condition of achromatism is not satisfied.
In the same way $d \neq (f_1-f_2)$ and hence the condition of minimum spherical aberration is not satisfied. But by making the total deviation to be shared among all the faces of the two lenses the condition for minimum spherical aberration is maintained. Also, using achromatic doublet as field lens and eye lens chromatic aberration is minimized

Cardinal points of a Ramsden eyepiece:

Equivalent focal length of the combination of the lenses [eyepiece] is given by,

$$F = \frac{f_1 f_2}{f_1+f_2-d} = \frac{f\times f}{f+f-\frac{2}{3}f} = \frac{3}{4}f \text{ - - - - } (1)$$

The position of the first principal point from the field lens is given by,

$$\alpha = \frac{F\times d}{f_2} = \frac{\frac{3}{4}f\times\frac{2}{3}f}{f} = \frac{f}{2} \text{ - - - - } (2)$$

The position of the second principal point from the second lens is given by,

$$\beta = -\frac{F\times d}{f_1} = -\frac{\frac{3}{4}f\times\frac{2}{3}f}{f} = -\frac{f}{2} \text{ - - - - } (3)$$

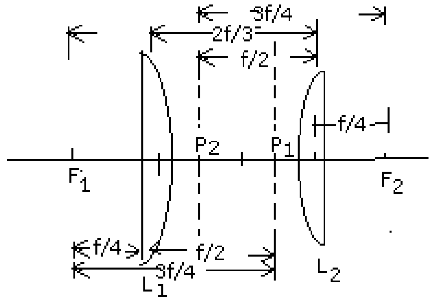


Figure (2)

Therefore the first principal point P_1 is at a distance of f/2 to the right of the field lens and the second principal point P_2 is at a distance of f/2 to the left of the eye lens. Since the system is in air, nodal points coincide with the principal points. The first focal point F_1 is at a distance of 3f/4 from the first principal point and hence it is at a distance of f/4 to the left of the field lens. Also, The second focal point F_2 is at a distance of 3f/4 from the second principal point and hence it is at a distance of f/4 to the right of the eye lens.

COMPARISON OF HUYGENS EYEPIECE AND RAMSDEN EYEPIECE:

Huygens Eyepiece	Ramsden Eyepiece
(1) The image formed by the objective is in between the field lens and eye lens and hence it is a negative eyepiece and hence cross wire cannot be used.	(1) The image formed by the objective is in front of the eyepiece and hence it is a positive eyepiece and hence a cross wire can be used.
(2) The condition for minimum spherical aberration is satisfied.	(2) The condition for minimum spherical aberration is not satisfied.
(3) It satisfies the condition of achromatism.	(3) It does not satisfy the condition of achromatism
(4) It is achromatic for all colours.	(4) It is achromatic for only two colours.
(5) It is used for qualitative purposes in microscopes and telescopes	(5) It is used for quantitative purposes in microscopes and telescopes.
(6) It has a negative power.	(6) It has a positive power
(7) The two principal planes are symmetrically situated with respect to the eye lens.	(7) The principal planes are symmetrically situated on either side of the eyepiece.
(8) It cannot be used as a simple microscope.	(8) It can be used as a simple microscope.